

Mathematical model of fluid injection in heavy oil reservoirs

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ABSTRACT

This paper describes the development of a mathematical model of a heavy oil recovery well, where a cylindrically shaped well is assumed. Several variables are considered, which primarily calculate the mass dispersion of an injected gas into oil, and also captures the essential physical phenomena associated with fluid transport and crude oil upgrading. The governing equations for diffusion/convection and Darcy's law are combined forming a partial differential equation system. The diffusion/convection equation represents the propagation of the injected gas fraction along a horizontal axis into a reserve of specific characteristics. Darcy's law drives this dispersion through a vertical reference axis. Molecular diffusion is considered the rate controlling step for absorption of the injected gas in heavy oils. Gas concentration in the heavy oil is dependent on time and distance from the well interface and is a function of its diffusion and dispersion coefficient in the porous medium.

Keywords: Heavy oil, oil sands, injection well, dispersion

1. INTRODUCTION

As conventional world light oil reserves decline, the production of heavy oil resources is predicted to increase significantly in the future. Since there is a dearth of experimental data on the fluid dynamic phenomena inherent in state-of-the-art technologies employed for heavy oil recovery, numerical modeling is one approach to optimize its extraction and the recovery of residual oil (Bravo et al., 2007, Butler et al., 1989a, 2000b, Cuthiell et al., 2003).

Kapadia et al. (2006) developed a model to determine butane dispersion, which is known to have much higher miscibility with heavy oil than CO₂. Due to butane's miscibility with heavy oil, their model assumes perfect miscibility. In reality, CO₂ as the Vapex gas, has a four-stage process whereby mixing takes place Jha, (1986). Thus, it is not feasible to consider these processes in the development of a simple model, since this phenomenon is highly complicated and not sufficiently well understood. However, it can be approximately modeled using an effective diffusion/dispersion coefficient. A lower value of diffusivity essentially means that the miscibility problems slow down the overall mixing and diffusion process. Thus the approach of Kapadia et al (2006) can be modified for CO₂ injection.

A model is currently being developed where a cylindrically shaped well is assumed, whereas, the model developed by Kapadia et al. (2006) assumes a rectangular shaped well. Several variables were considered for the development of this mathematical model, which primarily calculates the mass dispersion of the gas into the oil. Initially the inner surface (or wellbore surface) is saturated with the extraction vapor. This model uses an equilibrium boundary condition, where the saturation concentration of the Vapex gas is reckoned at equilibrium pressure. The model can be improved by employing a non-equilibrium boundary condition. The Vapex gas then

slowly diffuses into the porous oil and mixes with it. Thus, the mass fraction of vapor, ω , increases with time. This reduces the viscosity of the oil allowing it to be extracted. There is bulk flow of the live oil in both the r and z directions due to induced pressure difference and gravity respectively, see Figure 1. In this model, oil is extracted from the sides of the well.

This model entails two components of velocity – horizontal and vertical. The vertical component is affected by gravity only. The horizontal component is caused by the induced pressure difference between injection and extraction. CO_2 is injected at a high pressure, and live oil is extracted at approximately a vacuum. This pressure difference results in a pressure gradient in the radial direction, which results in a horizontal Darcy velocity. As a result, the actual velocity is decreased at some points by the high viscosity of heavy oil. The normal velocity is zero at solid boundaries. It is only non-zero at boundaries where extraction of oil takes place. As oil is extracted, the height of the well decreases with time.

The present model, thus accounts for the molecular distribution of the Vapex gas in the heavy oil, the mixing and the bulk flow of live oil. It also accounts for the variation in well height. However, it can be shown that the time scales for the diffusion of Vapex gas, and the extraction of live oil are vastly different i.e. the well becomes fully saturated with Vapex gas long before the well height begins to decrease significantly. The model can be simplified by working in two stages. The first stage considers the diffusion/dispersion problem with a fixed well height. Once the well becomes fully saturated, the Vapex concentration remains constant at the saturated value. Thus, there is no longer any need to keep solving the dispersion problem. The second stage only deals with the variation in well height with time. This will greatly simplify the model without compromising accuracy. This will also allow COMSOL to be used in the Vapex stage, since it would not have to deal with the moving boundary. This model assumes that the first stage occurs quickly compared to the pumping stage, which has a higher time scale. As a result, the model assumes fixed boundaries during the Vapex process. This further implies that there is no net velocity since the live oil has nowhere to flow until it begins to be pumped out of the well. In reality, when the entire well becomes saturated with the Vapex gas, less than 1% of the oil has been pumped out of the well. So this simplification is advantageous. Therefore, the Vapex process essentially becomes a diffusion/dispersion problem. The pumping problem can now be solved by various techniques, including known correlations. But the Vapex process is the main process of interest.

When steam is used as the Vapex gas, the wellbore surface may not have a constant or consistent steam concentration, because of “bulbs” of steam that form locally. Modeling this requires knowledge of complex multiphase phenomena which is out of the scope of a simplified model. That needs to be addressed in the development of a more complex model.

2. MODEL DEVELOPMENT

Figure 1 illustrates an injection well with cylindrical coordinates and boundary conditions used to develop a simple model in COMSOL. The model represents a porous medium saturated with heavy oil and bitumen within a cubical volume; in this volume a gas injector and recovery well are horizontally placed as practiced in the VAPEX process. The gas is injected and diffuses into the block and gets absorbed by the heavy oil and bitumen. In the development of this mathematical model several assumptions were made (Kapadia et al., 2006):

- The Vapex gas exits the well screen at constant temperature and pressure.
- The oil reservoir has uniform porosity and permeability.
- The mass fraction of gas at the exposed surface of porous medium is the saturated mass fraction at equilibrium.
- The dispersion of gas proceeds along the r -direction. The transfer of gas along z -direction is governed by the z component of Darcy velocity in the porous medium.
- The dispersion of gas incorporates molecular diffusion, the effects of surface renewal and augmentation, and any convective component along the r -direction.
- There is no mass transfer across the vertical face of the block on the left hand side at $r = R_i$, which is the wall of the recovery well
- Symmetry is assumed in the θ direction, thus allowing for a 2D treatment.

2.1 EQUATIONS

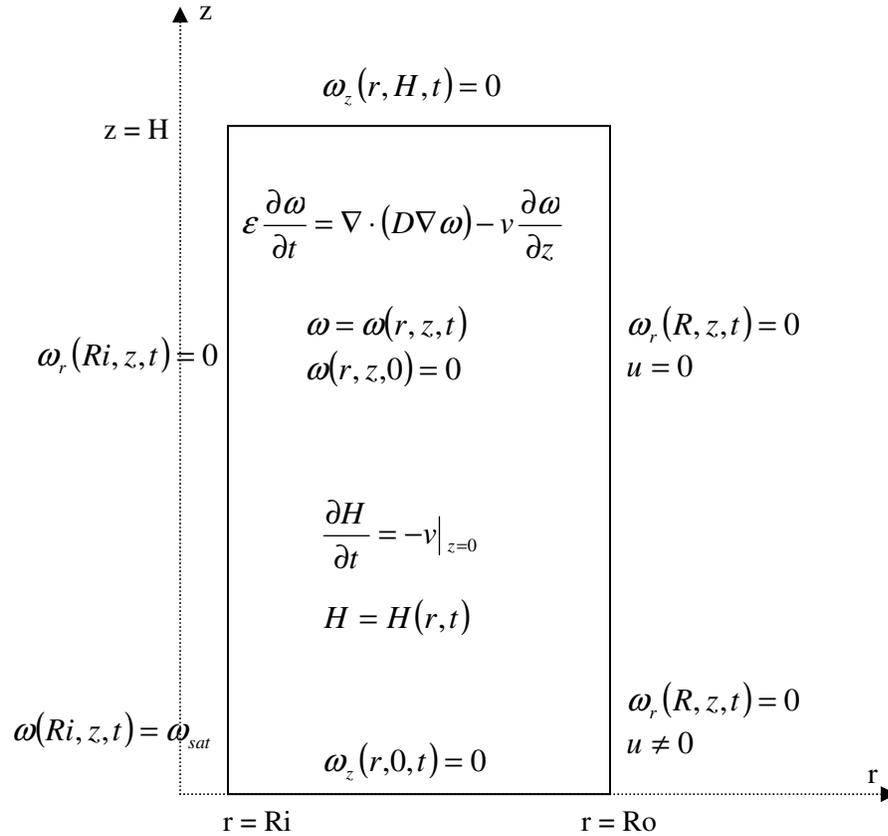


Figure 1: Schematic of cylindrical well (axis-symmetric).

Governing equation used for gas motion dispersion:

$$\varepsilon \frac{\partial \omega}{\partial t} = \frac{1}{r} \cdot \frac{\partial}{\partial r} \left(D \cdot r \cdot \frac{\partial \omega}{\partial r} \right) + \frac{\partial}{\partial z} \left(D \cdot \frac{\partial \omega}{\partial z} \right) - v \frac{\partial \omega}{\partial z} \quad (1)$$

Where:

ϕ = Porosity of medium.

ω = Mass fraction of gas in the block.

D = Dispersion of the gas along r direction

v = Darcy's law (represents mass fraction along xdirection).

$$v = \frac{K_r K_{rg} \rho g \cos \theta}{\mu} \quad (2)$$

Where:

K_r = relative permeability.

K = relative permeability of gas in the block.

ρ = density of live oil.

g = gravity.

μ = concentration viscosity/ dependency.

Where:

$$\mu = \mu_0 \omega^{-2} \tag{3}$$

For dispersion of gas dependency in heavy oil, we used:

$$D \propto \mu^{-0.5} \tag{4}$$

From equations (3) and (4) we can derive that the general dispersion dependency is equal:

Where:

$$D = D_0 \omega \tag{5}$$

D_0 = VAPEX gas diffusivity ($\omega = 1$).

These terms which apply to butane gas are defined and assigned numerical values in (Kapadia et al., 2006). These values and correlations have to be adjusted for carbon dioxide and other gases.

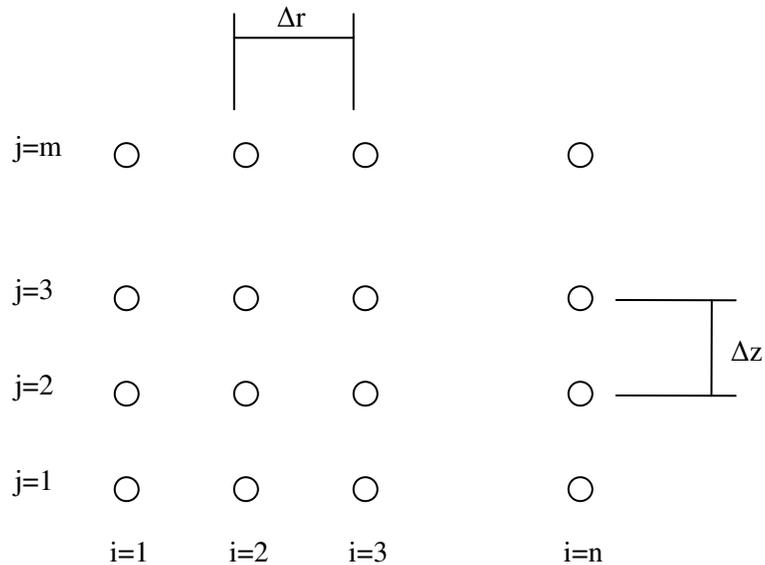
Since most injection wells are cylindrical, our model was adapted to cylindrical coordinates, where the governing equation becomes:

$$\varepsilon \frac{\partial \omega}{\partial t} = \frac{1}{r} \cdot \frac{\partial}{\partial r} \left(D \cdot r \cdot \frac{\partial \omega}{\partial r} \right) + \frac{\partial}{\partial z} \left(D \cdot \frac{\partial \omega}{\partial z} \right) - v \frac{\partial \omega}{\partial z} \tag{6}$$

$$= \frac{1}{r} \cdot \frac{\partial}{\partial r} \left(D_0 \omega \cdot r \cdot \frac{\partial \omega}{\partial r} \right) + \frac{\partial}{\partial z} \left(D_0 \omega \cdot \frac{\partial \omega}{\partial z} \right) - v_0 \omega \frac{\partial \omega}{\partial z} \tag{7}$$

$$= D_0 \left[\omega \frac{\partial^2 \omega}{\partial r^2} + \left(\frac{\partial \omega}{\partial r} \right)^2 + \frac{\omega}{r} \frac{\partial \omega}{\partial r} + \omega \frac{\partial^2 \omega}{\partial z^2} + \left(\frac{\partial \omega}{\partial z} \right)^2 \right] - v_0 \omega \frac{\partial \omega}{\partial z} \tag{8}$$

The domain is discretized into equally spaced nodes



The following central difference approximations are used:

$$\begin{aligned}\frac{\partial \omega}{\partial t}(i, j) &= \frac{\omega_{new}(i, j) - \omega(i, j)}{\Delta t} \\ \frac{\partial \omega}{\partial r}(i, j) &= \frac{\omega(i+1, j) - \omega(i-1, j)}{2\Delta r} \\ \frac{\partial^2 \omega}{\partial r^2}(i, j) &= \frac{\omega(i+1, j) + \omega(i-1, j) - 2\omega(i, j)}{\Delta r^2} \\ \frac{\partial \omega}{\partial z}(i, j) &= \frac{\omega(i, j+1) - \omega(i, j-1)}{2\Delta z} \\ \frac{\partial^2 \omega}{\partial z^2}(i, j) &= \frac{\omega(i, j+1) + \omega(i, j-1) - 2\omega(i, j)}{\Delta z^2}\end{aligned}$$

2.2 BOUNDARY CONDITIONS

Boundary value problems in mathematical model development are established by a set of partial differential equations that are governed by restraining boundary conditions (Polyanin and Zaitsev, 2003, Polyanin, 2002). Furthermore, this set of equations must satisfy the boundary values given at each domain. For multi-domain modeling, all boundary conditions must be stated at each sub-domain boundary and interface. For this model the initial boundary conditions are established as follows:

Primarily, there is only gas at the right vertical side of the block; this value of gas concentration is given by the initial states of pressure and temperature of the media (Kapadia et al., 2006). Therefore, the values of concentration at $t = 0$ are:

$$\omega = \begin{cases} 0, & \text{for } 0 \leq z < Z_0, 0 \leq r < R_0, \\ \omega_{sat}, & \text{for } 0 < z < Z_0, r = R_0 \\ Z = Z_0 \end{cases}$$

At $t > 0$ the entire block is exposed to the gas injection; the boundary conditions are established as:

$$\omega = \omega_{sat} \begin{cases} \text{for } 0 \leq z \leq Z, r = R_0; \\ \text{for } 0 \leq r < R_0, z = Z_0; \end{cases}$$

$$\frac{\partial \omega}{\partial x} = 0; \quad \text{for } 0 \leq z \leq Z \text{ and } r = R_i;$$

2.3 SOLUTION OF ALGORITHM

Fig. 3 shows the general solution algorithm used. Once the governing equations are established a numerical method is used to approximate a result that satisfies the initial and boundary conditions stated at each sub-domain of the model. Firstly, Fig. 1 shows the initial values of gas concentration for this propagation diffusion problem, and secondly new values of concentration are calculated at each grid node within this range.

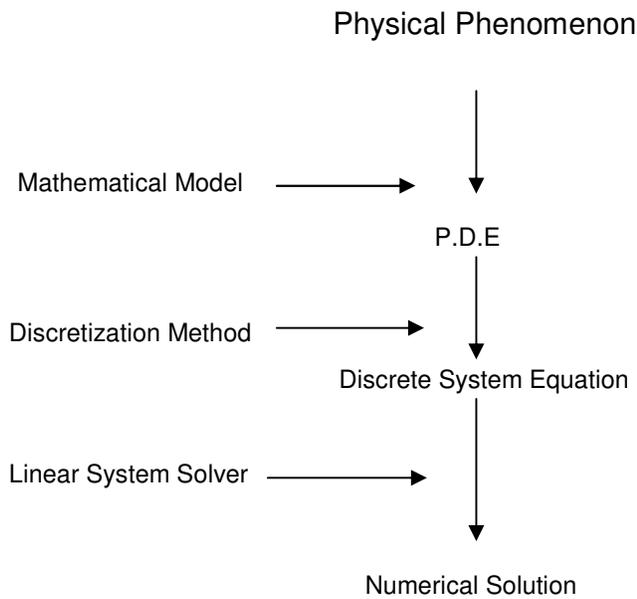


Figure 3: Schematic Algorithm

2.4 NUMERICAL VALUES

Table 1 lists the numerical values used in the computation of the model. In order to predict the diffusion behavior of different gases in heavy oil environments, some of these parameters can be easily modified as needed.

Table 1: Numerical Values used in the computation (Kapadia et al., 2006).

Parameters	Value	Units
Gravity (g)	9.81	m/s ²
Relative permeability (K _r)	1	
Permeability (K)	1.34 x 10 ⁻¹²	m ²
Density (ρ)	850	Kg/m ³
Angle of injection (θ)	π/4	radians
Viscosity (μ _o)	5.4709 x 10 ⁻⁴	Kg/m.s
Concentration (ω _{sat})	0.87	
Porosity (ε)	0.38	
Diffusion coefficient (D _o)	5.56 x 10 ⁻⁵	m ² /s

3. RESULTS AND DISCUSSION

This mathematical model is based upon the behavior of a gas diffusing into a heavy oil and bitumen environment. The solution at several points of the sub-domain symbolizes the values of CO₂ concentration in a cubical domain where the gas is being injected. Because of the simplifying assumptions made earlier we were able to implement this model for a horizontal well-bore into the multi-physics software, COMSOL. The chemical engineering module of this program is capable of generating a solution using a finite element mesh method and solutions can be presented as discrete values in a three dimensional form as shown in Fig. 6.

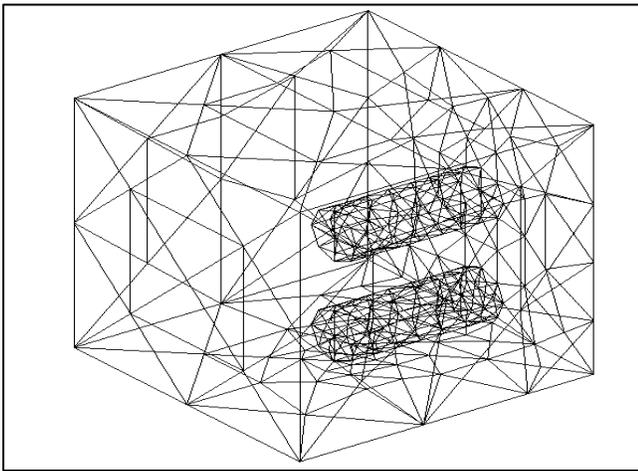


Figure 6: Mesh representation of domain

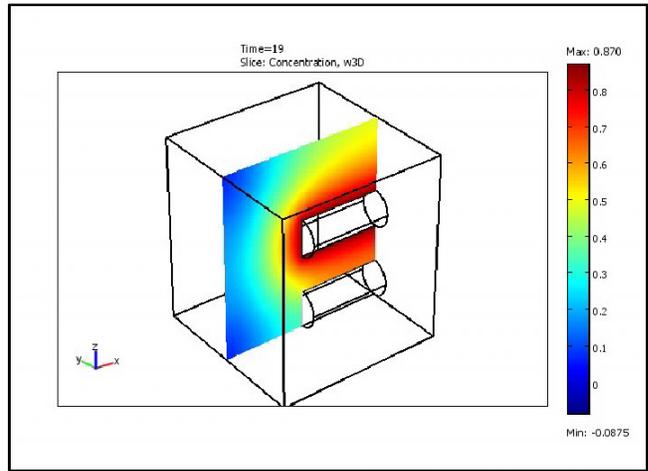


Figure 7: Domain concentration of CO₂ after 19 seconds.

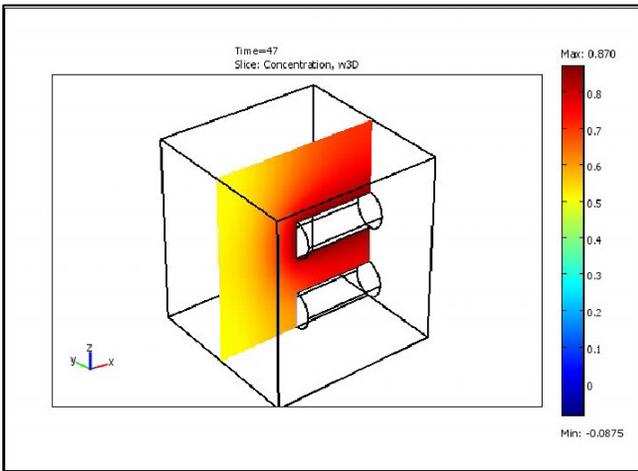


Figure 8: Domain concentration of CO₂ after 47 seconds.

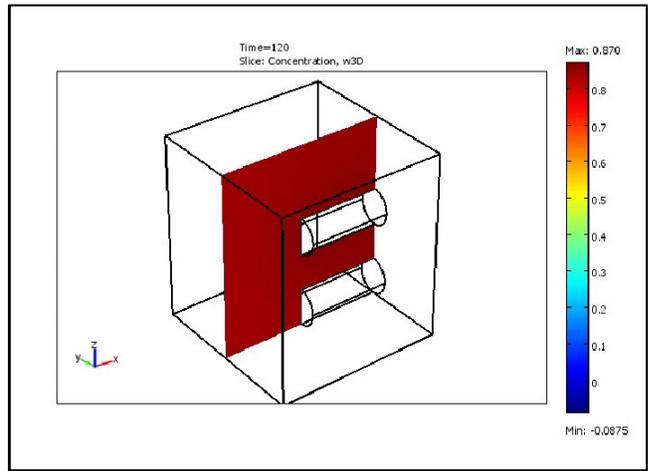


Figure 9: Domain concentration of CO₂ has reached steady state after 120 seconds

Figures 7, 8 and 9 illustrate cross-sections of CO₂ concentrations as a function of time; the minimum value given as -0.0875 represents a small margin of error due to the selected mesh size. The concentration of CO₂ as it approaches a steady state at any chosen location (Arc-length) within the domain is shown in Fig. 10.

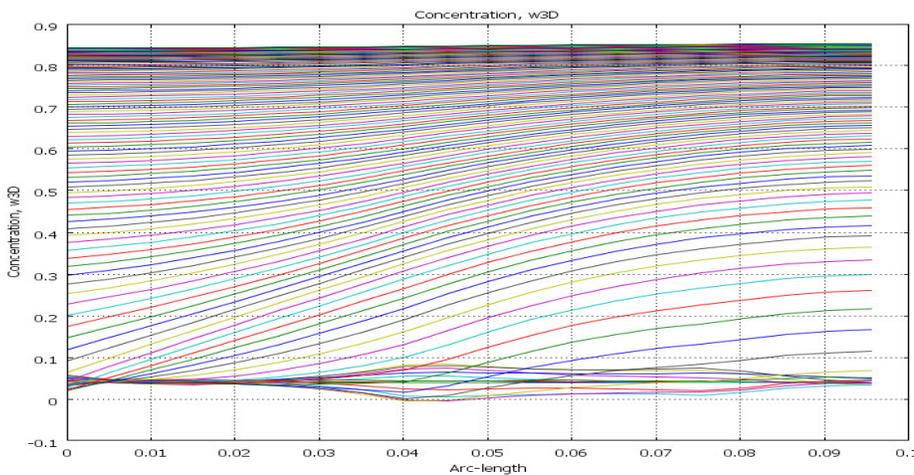


Figure 10: Concentration of CO₂ at a specific Arc-length of the sub domain as it reaches a steady state

4. CONCLUSION

A mathematical model was developed to simulate the dispersion and concentration of an injected gas into a homogeneous, porous heavy oil reservoir at a specific temperature and pressure. Several assumptions were made in the development of the model to represent the composition of the heavy oil and bitumen as well as to establish the behavior of the gas dispersion. The moving boundaries were also considered with respect to time in the development of this model. Some assumptions regarding boundary conditions were based on published results from other researchers using various gases. Additionally, cross-sections of gas concentrations are presented as a function of time. Finally, a post-analysis process has been developed, which enables the evaluation of the diffusion process up to a stage where the gas concentration reaches a steady state condition throughout the domain.

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